The Wave Theory of Numbers

Robert E. Grant[1], Talal Ghannam[2]
[1]Strathspey Crown Holdings, Crown Sterling. Newport Beach, California, USA.
[2]Crown Sterling, Newport Beach, California, USA.

Abstract
The concept of waves is very fundamental to classical and modern physics alike, being essential in describing light, sound, and elementary particles, among many other phenomena. In this paper, we show that the wave/particle duality is a phenomenon manifested not only in the physical world and the mathematics that describes it but also in the simple numbers that form the basic matrix upon which most of our sciences rest. We will also show how this wave-based approach to numbers could be essential to our understanding of the mathematical and physical constants that govern the physical laws as well as the natural elements emerging from them.

I. INTRODUCTION

Waves are one of the most studied natural phenomena and in most scientific fields. This is due to their ubiquitous presence everywhere in the world around us, from the macrocosmic to the microcosmic levels.
The most common form of waves is the one observed on water surfaces, mostly due to the mechanical forces that winds exert on surfaces like seas, lakes, ponds, etc. Simply drop a stone into still water and you will generate circular waves that expand outwardly from the center (where the stone was dropped).
Sound also propagates in the form of waves that travel by compression and expansion of air molecules[1].

Waves emanating from two or more sources interfere with each other creating a pattern of alternating light and dark regions, being the result of the constructive and destructive interference between the troughs and crests of the waves. When troughs meet troughs (or crests meet crests) they add up constructively. When a trough and crest meet they add up destructively, canceling each other out.
This pattern is one of the main proofs of the wave-like nature of light, as demonstrated by Young's famous double-slit experiment[2].

In fact, interference patterns are the hallmark of waves propagation and one of the most important tools used by scientists allowing them to measure important qualities such as the frequencies of the interfered waves, the speeds of their sources, the distances of these sources, among many other properties of interest.

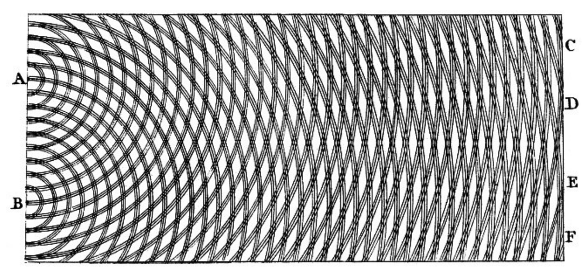


Fig.1: A sketch by Thomas Young illustrating light diffraction-interference pattern.

At the beginning of the 20th century, it became evident that the classical mathematical models used in physics for centuries are inadequate to explain the new observations and discoveries the experimental physicists were making.
The breakthrough came in 1913 when Niels Bohr, a Danish physicist, suggested a novel way to look at the orbits of the electron around the proton in the hydrogen atom; instead of a continuum of possible energy states, only discrete or quantized ones are allowed[3]. The success of his model in predicting the atomic spectrum of the hydrogen

atom was the spark that ignited the quantum mechanics revolution.

Another breakthrough came from the French physicist Louis De Broglie who suggested that particles may also behave like waves inside the atoms; exhibiting wavelengths and frequencies just like sound and light do[4].

This theory explained many observations and opened the door into an exciting new reality that is, however, not easy to comprehend nor to visualize[5].

The wave-particle duality was confirmed in 1927 when a different type of Young's double-slit experiment was performed, the Davisson-Germer[6] experiment, where interference diffraction patterns were observed from a stream of electrons (instead of light.)

Previously, in 1905, Albert Einstein had suggested in his photo-electric paper[7] that light is made of discrete wave packets of energy called photons. Thus, light also behaves either like a wave or as a point-like entity, depending on the observation or the experiment performed.

Consequently, the term *particle/wave duality* was coined to describe this mysterious phenomenon.

But where does this duality come from? And to what level do its ramifications reach? We definitely can't feel it in our macrocosmic level. Maybe we need to go to smaller levels to detect its origin, smaller even than the fundamental particles themselves.

There are many levels below that of the elementary particles. For example, protons and neutrons are believed to be made of even smaller entities called quarks, three of them in each[8]. And in String Theory[9], everything, including quarks, is supposed to be made of tiny vibrating strings, which is the most fundamental level physicist have theorized.

But there is still a more fundamental level; that of the physical and mathematical constants which govern space and natural forces, such as Pi, Euler number, fine structure constant, to mention a few.

II. THE PARTICLE-ASPECT OF NUMBERS

The idea behind atoms goes way back to the 5th century BC, where the Greek philosopher Leucippus[10] thought matter to be continuously devisable until one reaches a point where it cannot be divided anymore: the size of atoms.

By the beginning of the twentieth century, atoms were found to be made of even smaller entities, called elementary particles, mainly the electron, the proton, and the neutron.

Consequently the field of quantum mechanics emerged whereby physicists had a mathematical framework able to describe these particles and their interactions.

By assigning specific numbers to these particles, called *Quantum Numbers*[11], physicists were able to describe their quantum states. These quantum numbers are conserved and dictate how the particles behave and whether a certain reaction is allowed or not.

For example, in the beta-decay, where a neutron decays into a proton, an electron, and an antineutrino, the quantum charge number should be conserved before and after the interaction:

$$n^0 \rightarrow p^+ + e^- + \bar{\upsilon}\ .$$

And we see that 0 = 1 – 1 + 0, as the proton is considered positive, the electron negative, and the antineutrino carries no charge, just like the neutron.

There are many other qualities assigned to particles, such as angular momentum, principle quantum number, spin, etc. and most of them are described by sets of integers either discrete (i.e. similar to [-1, 0, 1] as in the charge number for example) or continuous, as in the principle quantum number [1, 2, 3...]

But we are not constrained to use these specific numbers only. We can use numbers from 1 to 9 to describe the elementary particles and their behaviors, requiring the implementation of the digital root math[12].

For example, if we assign particles the numbers shown in the table below, we find that these numbers also satisfy the beta-decay, and any elementary reaction, as follows: 1 = 2 + 3 +5 = 10 and the digital root of 10 = 1.

PARTICLE	ANTI-PARTICLE
$\boldsymbol{n^0} \rightarrow \mathbf{1}$	$\overline{n^0} \rightarrow 8$
$\boldsymbol{p^+} \rightarrow \mathbf{2}$	$p^- \rightarrow 7$
$\boldsymbol{e^-} \rightarrow \mathbf{3}$	$e^+ \rightarrow 6$
$\boldsymbol{v} \rightarrow \mathbf{2}$	$\bar{v} \rightarrow 2$

The numbers assignment is chosen such that each particle, when added to its antiparticle, results in the number 9, which is equivalent to 0 in the digital root math. However, number 9 is not 0 or nothingness, and this is where the two systems differ; as when a particle and antiparticle unite, they annihilate each other in a great burst of energy. This is exactly what number 9 is telling us here: that the unification results in energy, not 0 (as the numbers 1 and -1 produce).

Therefore numbers can enact the role of particles perfectly (and energy, as in number 9). They even dictate how particles interact and behave. In a sense, it is from the properties of numbers that the physical reality inherits its fundamental laws.

III. THE WAVE ASPECT OF NUMBERS

As illustrated above, particles and numbers are different manifestations of the same point-like aspects.

However, as we have shown above, this point-like aspect is not the only one that particles exhibit; they have a wave-like nature that is as fundamental as the point-like one, described by wave-equations, such as Schrödinger's equation[11], interpreted as a probability density spread over spacetime, communicating a physical meaning only when squared.

There are two main types of numbers: integers and non-integers (floats).

Integer numbers are exact, with no commas or leftovers. So, an integer would be something like 1, 2, 3, 564, 9845603, etc.

By contrast, non-integer or float would be something like 5.65, or 2.7182818…

For the case of floats, there are several types. There are those that are rational, having few numbers for their remainders or where the remainder repeats indefinitely in a specific pattern, like 1/7 = 0.142857142857…

And there are irrational numbers, being those numbers where their remainders repeat in a completely random fashion, but still are the solution of some algebraic equation, such as the golden section Phi[13].

On the far extreme, we have transcendental numbers, where their remainders repeat indefinitely in a string of random digits. Unlike transcendental numbers, however, they solve no known algebraic equation, such as π or Euler number *e*.

The ratio 1/7 is particularly interesting as it seems to capture the essence of the wave. This is because the repeated pattern of its remainder [142857] mimics a wave when plotted individually, and more so collectively, as shown in figure (2) below.

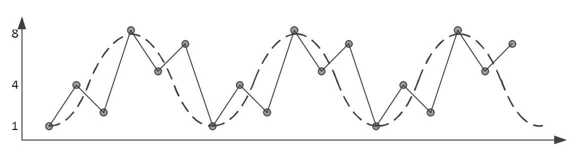


Fig.2: The wave-like nature of the remainder of 1/7.

In fact, taking the inverse of numbers is the simplest mathematical way to create repeated patterns, for example, 1/6 = 0.16666…, or 1/273 = 0.00366300366300… etc.

Natural numbers [0, 1, 2, … ∞] only increase in magnitude and, consequently, no wave-like behavior is observed. However, looking at their digital roots exposes a wave-pattern embedded underneath.

So, when numbers from 1 to infinity are distributed, let's say within three columns, we find that the digital roots of the numbers in each column repeats indefinitely, as shown in the table below.

NATURAL NUMBERS			DIGITAL ROOT		
1	2	3	1	2	3
4	5	6	4	5	6
7	8	9	7	8	9
10	11	12	1	2	3
13	14	15	4	5	6
16	17	18	7	8	9
…	…	…	…	…	…

The 1st column's digital root repeats in the sequence [1, 4, 7]. This sequence resembles a wave, just like 1/7 does. It's the same for the other two columns made up of [2, 5, 8] and [3, 6, 9].

Therefore, in principle, numbers do observe some form of a numerical wave-like pattern. We can say that numbers carry certain frequencies, either individually, as in their repeating remainders, or collectively, as in their digital roots.

If this wave-like essence is related to the same one we observe everywhere in nature, it should also exhibit some of its properties. And there is no property of waves more fundamental than interference.

IV. NUMBERS' INTERFERENCE AND PATTERNS

One of the most basic interference patterns is the one generated from two point-like sources propagating circular waves. The emerging pattern is similar to that of the two-slit experiment (Figure 1) reproduced in greater detail below:

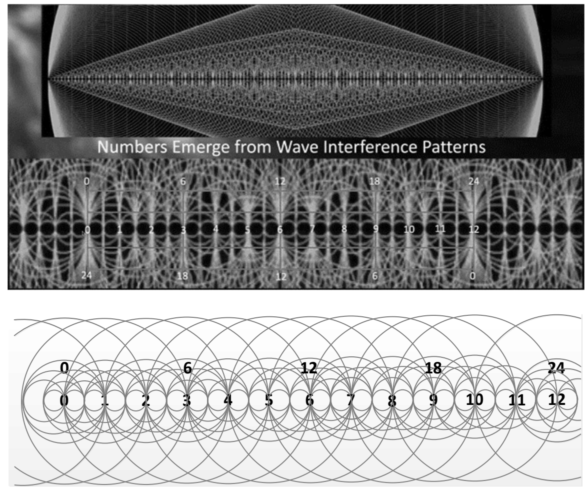


Fig.3: (top) Interference pattern from two point-like sources. (Middle) The same interference pattern magnified to illustrate the 12-based fractal nature of the pattern. (Bottom) Digital rendering of the interference pattern for a scale from 0 to 12.

From the above pattern, we notice that the waves' interference creates periodic self-similar fractals with intervals obeying multiples of 12, depending on the level of the fractal. Moreover, the pattern is built on doubling principles where each circle encompasses an even number of smaller circles inside it in a hierarchal fashion.

On closer inspection (Figure 4), we see that the numerical wave matrix is scalable to numbers from 1 to 10 as well as from to 100, or to 2000, etc. For our case, we look at a range from 1 to 121 for reasons explained shortly.

The numbers on the horizontal axis are referenced by their digital roots instead of their original full values for better illustration of the patterns of the wave matrix.

Note how the matrix is symmetric around number 60, especially when it comes to prime numbers.

Prime numbers are divisible by themselves and number 1 only. The Fundamental Theorem[14] of Arithmetic states that any integer >1 can be expressed as the unique product of two or more primes. In this sense, prime numbers can be considered the main block upon which all other numbers are built, giving them a special status among the rest.

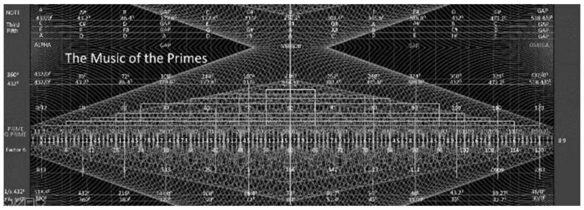


Fig.4: An illustration of the wave matrix generated from two-point-like interfering sources and for a scale between 0 and 121, along with prime and Quasi-prime symmetry, angular references, and musical notes correspondences. (Please refer to Appendix A for a more detailed explanation.)

So, starting with number 60, the two numbers bounding it (59 and 61) are both prime numbers. Going further to numbers 54 and 66, we find another symmetry emerging with 53 and 67 being primes and 55 and 65 not. (We label these non-prime numbers Quasi Primes, being the product of prime numbers that are ≥ 5, thus excluding 2 and 3. Please refer to reference 15 for more information on these numbers[15].)

Numbers 47 and 49 to the left skip their matching pair and match up instead with the next pair of 77 and 79, while 71 and 73, both prime, match up with 41 and 43, also primes. These four pairs form an offset that creates a kind of momentum around number 60.
Further out, we get the symmetry back with pairs 35 and 37 matching up with 83 and 85.

When we reach numbers 30 on the left and 90 on the right, we lose our reflection symmetry for the pairs [29, 31] and [89, 91]. This is because, at 90 we have reached the fractal limit of our 9-base system, starting with 9, then 90, then 900, etc. Therefore, the break in symmetry here indicates the end of one cycle and the beginning of another. (We are working in a 9-based system not a 10-based one because 10 is reducible to 1 in the digital root math, and hence we have 9 numbers only, from 1 to 9.)

One other cycle is at number 24, where twin primes appearance on both sides of numbers that are multiples of 6 breaks and the first quasi-prime number appears, being number 25.
At 114 we get another break of symmetry, paving the way for the appearance of the first quasi-prime pair of [119, 121] and with the cycle finishing at 120, as in a full octave.

With the 12-based fractal of the wave matrix, we can superimpose the 12 notes of the octave on the pattern as if the interference is generated by sound waves, as shown in figure (4) above.
We used the notes of the major 5^{th} octave with the note A_5 tuned to the standard pitch of 432Hz. This is because by using this tuning value, we get a perfect correspondence between the angular reference of the notes' and the internal angles of specific polygons as explained in detail in reference 16.

Therefore the wave interference pattern is a unifying matrix where numbers, as well as geometry and sound, can all be manifested and expressed, enabling us, in return, to find new connections between these various disciplines, from music to the fundamental elements of nature.

V. WAVE INTERFERENCE AND THE EMERGENCE OF FUNDAMENTAL CONSTANTS

The geometry of the wave matrix is essentially a numbering or a measuring system which determines the scale at which the physical qualities that emanates from these waves work at.

For example, if we to consider these waves of electromagnetic nature, emanating from the fluctuation of the zero-point energy[17], then their interference pattern would set the scale that controls many physical laws and qualities, such as the speed of light. In another non-vacuum medium, the scale will change depending on the structure of the medium and the speed of light will change accordingly.

We can also think of each point on the wave matrix as an individual number-source generating its own propagating numerical waves. This logic is synonymous with the Huygens–Fresnel Principle[18] which is very powerful in explaining many properties of light that are not explainable using the single source model alone.

Therefore, mathematical constants, such as π or e, can be decomposed into specific integers plus a wave. For example, the constant $\pi = 3.14592\ldots$ is partitioned into 3, representing the particle aspect of the number or its amplitude, plus a remainder of 0.141592…

To make the concept more comprehensible, we use the approximate ratio of $22/7 = 3.142857\ldots$ which is equivalent to 3 (amplitude) + 1/7 (phase). The same logic applies to Euler number $e = 2.718\ldots$ which can be expressed as $19/7 = 2.714285\ldots = 2 + 5/7$, with 2 being the amplitude and 5/7 being the phase.

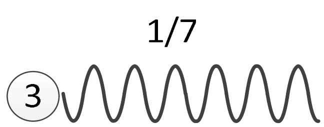


Fig.5: Number π expressed in a wave/particle fashion; the sum of an integer amplitude of 3 and a phase of 1/7.

In this respect, each number on the wave matrix form a seed or a kernel around which mathematical and physical constants grow through the addition of a phase.

By deducing the mechanism through which the phases are generated, we can start predicting the values of other constants that are not yet identified. This is being investigated at the moment and will be discussed in the next paper.

The above reasoning invites a more geometrical approach to the mathematical and physical constants, being *natural separations of the waves*.

For example, number π emanates from the wave-separation of unity in the ratios of 3 and 4 as illustrated in the image below to the left. The remainder can be generated from the same process; endless doubling or separation of the circular waves. And so is for the rest of the mathematical constants; emerging from the overlapping or separation of propagating waves.

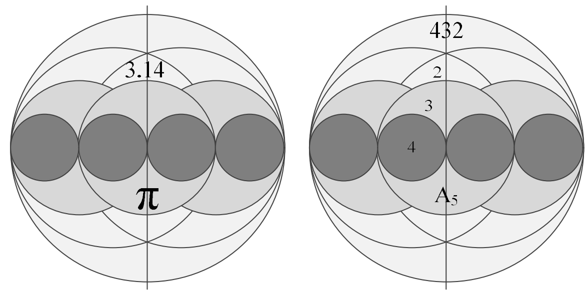


Fig.6: Mathematical constants represented as doubling of waves/circles: for π (left), and for the standard tuning of 432Hz (right).

This is a powerful representation that transforms the abstract numerical constants into more comprehensible forms having unique wave-like signatures. This could very well be how these constants are communicated to the universe as well as the mechanism by which it governs its physical laws

VI. THE PERIODIC WAVE OF ELEMENTS

The natural elements comprising our universe are listed in a linear fashion in the Periodic Table of Elements, first introduced by the Russian chemist Dmitri Mendeleev in 1869. This table of 14-18 columns or families categorize the elements based on their atomic number (the number of protons in the nucleus) and their chemical properties.

Applying the wave theory to the numerical references of the elements (i.e. their atomic numbers) will separate these elements into their respective families more naturally than the linear method does.

As shown in figure (7) below, we start with hydrogen, the simplest element, forming the first wave/circle. (For better visualization, please refer to Appendix B.)

Hydrogen initiates the binary doubling of the first wave into 2 smaller ones, followed by 4 and then 8. These 8 circles are superimposed sine/cosine waves with one wave corresponding to the elements from He to Ne, while the other corresponding to elements from Ne to Ar. (Some elements appear on both sides of the wave matrix because it is a fractal configuration; when one cycle finishes, an exactly similar one starts.)

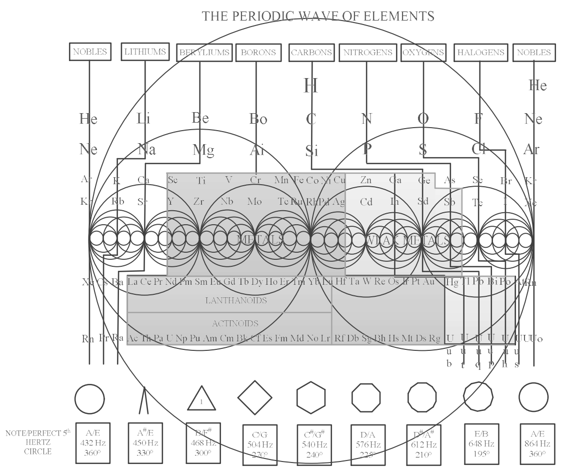


Fig.7: The wave-based table of elements along with their respective families (shaded areas) and their musical notes/geometry correspondences. (Please refer to Appendix B for a more detailed explanation.)

By continuing the octave doubling, elements line up perfectly in a manner that reflects their respective families, such as Metals, Lanthanoids, Acteonids, etc. with noble gases positioning themselves on both sides of the wave matrix, being the chemically inert members of the elements. The more we move closer toward the center or the peak of the wave the more active and isotopic the elements become.

Next comes carbon whose family obeys the following sequence: Si, Ge, Sn, and finally Pb.

Each element is positioned at a specific point or node that is determined by the interference of two waves. These waves emanate from the doubling of the previous one, representing the preceding element that initiates the doubling and the emergence of the next octave elements, and so on.

Recalling how the wave matrix corresponded to musical notes and geometry, we can therefore associate each element with a specific frequency and shape.

So for carbon, it corresponds to note $C_{\#}$ of 540Hz as well as to the hexagon, which is a very appropriate shape as carbon chains usually take the form of hexagons in organic materials.

The correspondences of the rest of the elements are illustrated in Appendix B.

A consequence of the above wave-doubling configuration concerns the counter-intuitive conclusion of missing elements on the hydrogen level (or even above this level) such that when doubled, will generate the next octave of the carbon level.

By following this logic to its narrowing conclusion, 19 extra elements (3 noble gases, 8 standard elements, and 8 isotopes) need to be added to the already known 118 of them, totaling to 137, the same number that Richard Feynman, the famous American physicist, already suggested[19] for the total number of elements (and also a well-known prime value encoding the fine-structure constant.)

Whatever the nature of these suggested new elements may be, if any, the wave-based configuration introduces a novel perspective to the elements and their families that may lead to a better understanding of their numerical construction as well as their geometrical and acoustic quintessence.

IV. CONCLUSION

Integrating the wave/particle aspect with integers and their fractions creates a novel and holistic approach to the physical world.

Treating numbers as natural wave generators, as well as separators, sets a scale which extends our reach into the most fundamental levels of many branches of science and thus provides us with the necessary knowledge to fathom the intricate relationships between these branches and to discover some of their hidden peculiarities.

APPENDIX

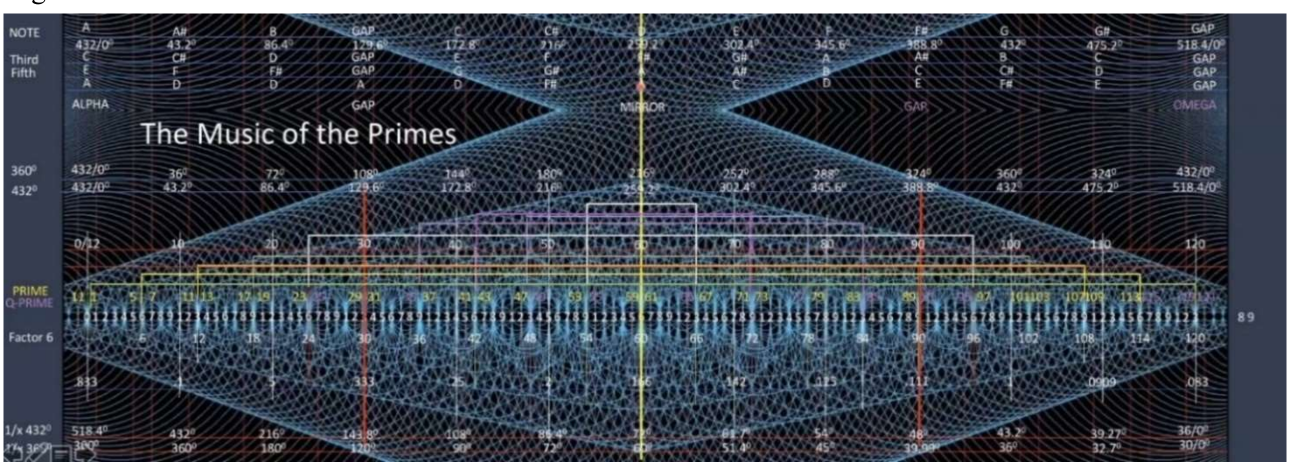
A- Larger rendering of Figure (4) with further explanation.

On the main horizontal axis of the wave interference matrix, we find the digital roots of numbers from 1 to 121. listed on top of them primes and Quasi-primes numbers positioned in a symmetrical pattern around 60 as explained above. (Number 60 can also be 6, 0.6 or 600, etc. as the whole pattern is of fractal nature.) Above the level of prime numbers, we have numbers' angular references as a fraction of 360° and 432°. So, above 60 we have $0.6\times360^\circ = 216^\circ$ while 288° is just $0.8\times360^\circ$ and so on. We use the decimal fraction of the numbers as to maintain the degree references within the 360° circle.

Notice how the cycle ends at 120, corresponding to 432°, which is the same value for the Pythagorean tuning.

The axis below the digital root numbers is the $1/x$ reference of the numbers along with their degree references. So, $1/6 = 0.166\ldots$ and $0.166\times432^\circ = 72^\circ$, and so is the case for the rest of the numbers (the $1/x$ analysis is essential for our next paper where we discuss the emergence of the fundamental constants from the wave matrix).

The top rows are reserved for the corresponding 12 musical notes and their degrees of reference.

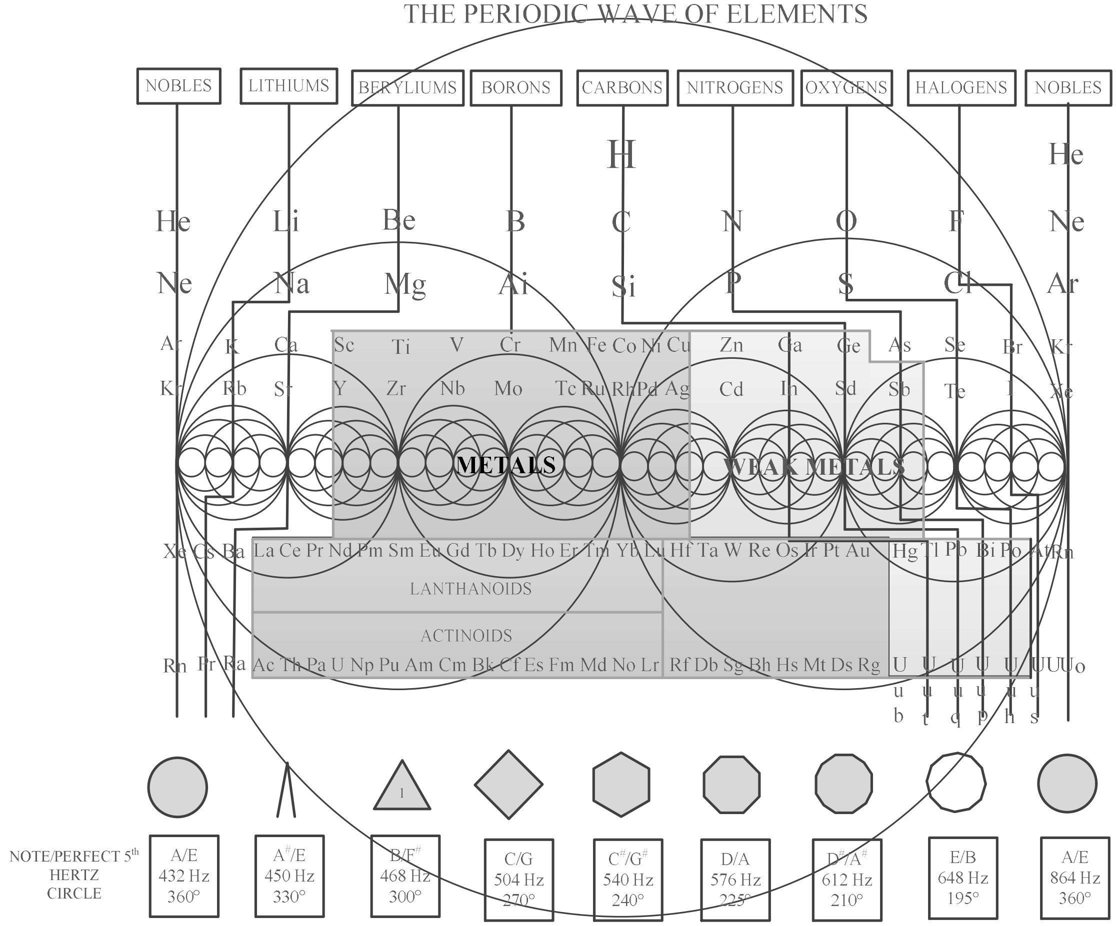


B- Larger rendering of Figure (7) with an explanation.

The wave-based periodic table of elements rests on the wave-doubling principle, with each element initializing the doubling of the wave, which in returns, produces more elements that initiate more doubling and so on. Each node corresponds to specific elements along with a specific musical note and geometry, as shown at the bottom of the image.

The shaded areas represent the chemical families of the elements, such as metals, weak metal, lanthanoids, and actinoids.

THE PERIODIC WAVE OF ELEMENTS



References

[1] David R. Dowling. Revealing hidden information with quadratic products of acoustic field amplitudes. Phys. Rev. Fluids 3 (2018).

[2] Young, Thomas. Bakerian Lecture: Experiments and calculations relative to physical optics. Philosophical Transactions of the Royal Society. 94: 1–16 (1804).

[3] Niels Bohr. On the Constitution of Atoms and Molecules. Niels Bohr, 1913-2013. Springer (2013).

[4] P. Weinberger. Revisiting Louis de Broglie's famous 1924 paper in the Philosophical Magazine. Philosophical Magazine Letters (2006).

[5] Feynman, R. QED: The Strange Theory of Light and Matter. Penguin (1990).

[6] Davisson, C. J.; Germer, L. H. Reflection of Electrons by a Crystal of Nickel. Proceedings of the National Academy of Sciences of the United States of America. 14 (4) (1928).

[7] Einstein, A. On a Heuristic Point of View about the Creation and Conversion of Light. Annalen der Physik 17 (1905).

[8] B. Povh, C. Scholz, K. Rith, and F. Zetsche. Particles and Nuclei. Springer (2008).

[9] Michael B. Green, John H. Schwarz, Edward Witten. Superstring Theory. Cambridge University Press (2012).

[10] C.C.W. Taylor. The Atomists: Leucippus and Democritus. Phoenix Presocractic Series (2010).

[11] Eugen Merzbacher. Quantum Mechanics. Wiley, 3rd edition, (1997).

[12] Talal Ghannam. The Mystery of Numbers: Revealed through their Digital Root, 2nd Edition. CreateSpace (2012).

[13] John Michell and Alan Brown. How the world is made. Inner Traditions (2009).
[14] Lindemann, F. A. The Unique Factorization of a Positive Integer. Quart. J. Math. 4 (1933).
[15] Robert E. Grant and Talal Ghannam. Accurate and Infinite Prime Prediction from Novel Quasi-Prime Analytical Methodology (2019).
[16] Robert E. Grant and Talal Ghannam. Music: The Unification of Sound, Numbers, and Geometry (2019). (Awaiting Journal Aproval).
[17] Harold E. Puthoff. Source of Vacuum Electromagnetic Zero-Point Energy. Physical Review A 40(9):4857-4862 (1989).
[18] David A. B. Miller. Huygens's Wave Propagation Principle Corrected. Optics Letters 16 (1991).
[19] Hans Eggenkamp. The Geochemistry of Stable Chlorine and Bromine Isotopes. Springer (2014).