

# Four-Fold Mirror-Symmetry Inherent to the Icositetragon Distribution of Numbers

Robert E. Grant<sup>1</sup>, Talal Ghannam<sup>2</sup>

<sup>1</sup>Strathspey Crown Holdings, Crown Sterling, Newport Beach, California, USA.

<sup>2</sup>Crown Sterling, Newport Beach, California, USA.

## Abstract

Recently, a 24-based distribution of numbers was used in a novel method for an infinite and accurate prime prediction and factorization. Therefore, further investigation of this particular configuration of numbers, labeled here the *icositetragon wheel*, is essential if we are to expand our understanding of this method and to further improve it. We will show that using the icositetragon wheel is not an arbitrary choice by elaborating on the unique properties this configuration has, not only in regard to prime numbers distribution, but also for the many symmetries and complementary properties that numbers, prime and non-prime, observe in it.

## I. INTRODUCTION

It has been recently shown that by exploiting the geometry of the 24-based distribution of numbers<sup>1</sup>, we can, in principle, predict prime numbers up to infinity and with high accuracy.

As shown below, using the 24-sided icositetragon as the grid, prime numbers occupy eight moduli only, called *prime moduli*, in the shape of a forked cross<sup>2</sup> (4 arms =  $24/6$ ).

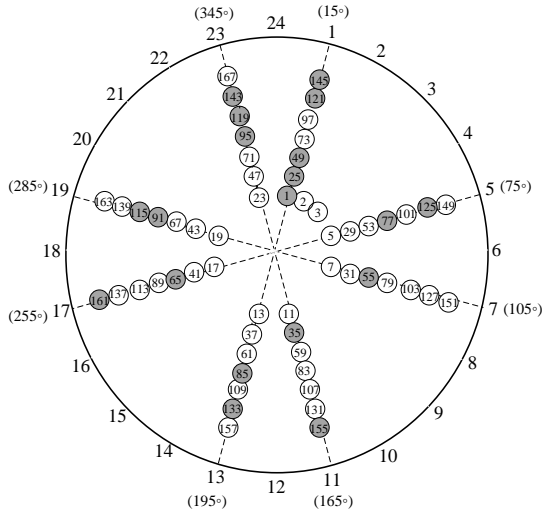


Fig.1: Prime numbers distribution around a 24-sided polygon or icositetragon wheel making a forked cross. (For a more detailed image, please refer to Appendix C)

Those numbers that are not prime, while at the same time occupying the prime moduli, are also

unique because they are the product of primes larger than or equal to 5 and/or semiprimes only. They are labeled Quasi-prime as to distinguish them from Semi-prime numbers<sup>3</sup>, which are the product of any two prime numbers, including 2 and 3.

As Quasi primes reside along the prime moduli only, we can therefore set up a multiplication grid, labeled the Q-grid, where the principal axes are made of prime and semiprime numbers only. Consequently, all the numbers within this Q-grid exist along the prime moduli while not being prime themselves. This property enables the prediction and testing of the primeness of any number by comparing the numbers in the Q-grid with those on the prime moduli of the icositetragon wheel<sup>1</sup>.

One reason the icositetragon configuration pertains particularly to prime numbers is the fundamental property where the square of any prime number is always equal to a multiple of number 24, plus 1. (For the proof, please refer to Appendix B.)

In this paper we further explore this distribution of numbers by analyzing their numerical configurations so as to identify any relationship or property that is a consequence of its geometry

and thus may enhance its capability of prime prediction and factorization.

## II. SYMMETRY BREAKING AND COMPLEMENTARY PROPERTIES

By projecting the numbers of the eight prime moduli onto the Q-grid, shown below, we discover some interesting behavior.

	5	7	11	13	17	19	23	25	29	31	...
5	25	35	55	65	85	95	115	125	145	155	...
7	35	49	77	91	119	133	161	175	203	217	...
11	55	77	121	143	187	209	253	275	319	341	...
13	65	91	143	169	221	247	299	325	377	403	...
17	85	119	187	221	289	323	391	425	493	527	...
19	95	133	209	247	323	361	437	475	561	589	...
23	115	161	253	299	391	437	529	575	667	713	...
25	125	175	275	325	425	475	575	625	725	775	...
29	145	203	319	377	493	561	667	725	841	899	...
31	155	217	341	403	527	589	713	775	899	961	...
...	...	...	...	...	...	...	...	...	...	...	...

Notice how those Quasi-primes belonging to the 1<sup>st</sup>, 4<sup>th</sup>, 5<sup>th</sup>, and 8<sup>th</sup> prime moduli continue along the grid without interruption (black lines), while those of the 2<sup>nd</sup>, 3<sup>rd</sup>, 6<sup>th</sup>, and 7<sup>th</sup> moduli are interrupted either by each other or by the other four prime moduli (dashed lines).

Thus, there is a sense of symmetry breaking between the horizontal and vertical prime moduli that is not obvious by considering the geometrical symmetry of the icositetragon wheel.

This is not the only symmetry-breaking observation we will observe.

By examining the northern, eastern and western central axes of the wheel, we find an interesting complementary property.

Aided by the figure below, we see that east and west moduli numbers add up to every other number on the *northern* modulus only.

For example, numbers 6 and 18 on the horizontal axis add up to 24 on the vertical axes,  $30 + 42 = 72$ ,  $54 + 66 = 120$ , and so on.

The southern modulus is excluded from this property, which is another asymmetric behavior.

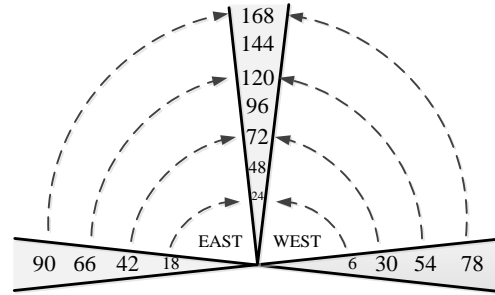


Fig.2: East-West central moduli add up to produce every other number on the northern central modulus.

Additionally, the northern and southern central moduli complement each other where two numbers on these moduli add up to produce a number on the southern one only as illustrated below. So, on the vertical axis we find:  $24 + 12 = 36$ ,  $72 + 60 = 132$ , etc.

The East-West central moduli, on the other hand, don't exhibit the same behavior.

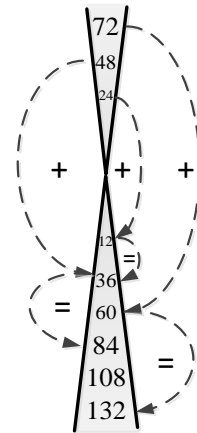


Fig.3: North-South moduli complementary property.

Below is an illustration of another complementary property where numbers on two prime moduli add up to another on the central moduli. For example, prime numbers 77 and 79 add up to 156, which lies on the southern central modulus. Also,  $12 = 5 + 7$ ,  $24 = 13 + 11$ ,  $24 = 23 + 1$ ,  $36 = 19 + 17$ ,  $60 = 29 + 31$ , etc.

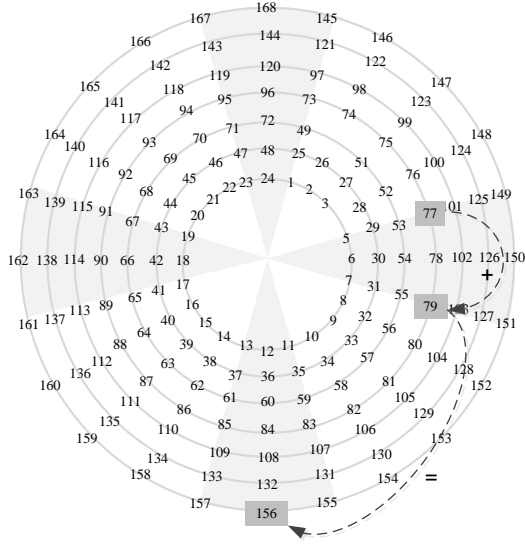


Fig.4: Prime numbers on the prime moduli adding up to numbers on the central moduli.

#### IV. QUADRANT SYMMETRY OF THE ICOSITETRAGON

It is obvious from the above analysis that the icositetragon numerical configuration has a quadrant nature.

These quadrants are defined by the main horizontal and vertical central moduli that, at the same time, work as complementary axes where the numbers on the central moduli possess complementary relationships with numbers on other central moduli (Figures 2 & 3), as well as with numbers belonging to the prime moduli (Figure 4) as explained above.

This quadrant behavior is something we encounter in many fields of physics and mathematics, such as in the complex plane where the real and imaginary axes define a similar quadrant configuration, with the quadrants on the negative side often being a continuation or reflection of those on the positive side, as proposed by the complex function continuity theorem<sup>4</sup>.

Intrinsic to any coherent quadrant configurations is the existence of some form of relationship or symmetry between the elements of each

quadrant, i.e. having a reflection symmetry around the horizontal and/or vertical axes.

This is also the case for the numeric icositetragon wheel, as each integer along the central moduli, along with the prime moduli that surround them, possesses *circular-complementary* relationships with other numbers along these same moduli such that they add up to 360 (or multiples of 360) and are therefore parts of the same quadrant integer set.

Figure (5) below explains this circular relationship between the four members of a quadrant set.

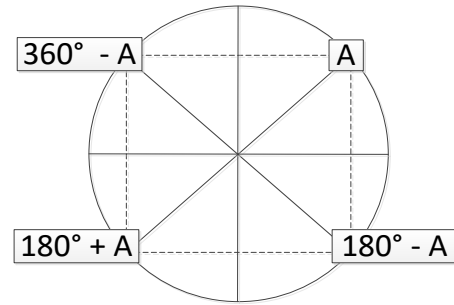


Fig.5: Definition of the circular complementary relationship between a set of four numbers with A being one member of the set.

Let us use one example to explain the above argument.

Taking number 341 on the 5<sup>th</sup> modulus, its complementary number 19 is found in the opposite mirrored 19<sup>th</sup> modulus ( $5+19 = 24$ ), when reflecting across the central vertical moduli (24 and 12). This is because  $341+19 = 360$ . The two additional complements, 161 and 199, are reflected through the horizontal moduli (moduli 6 and 18).

These two numbers are part of the same quadrant set not only because  $161+199 = 360$ , but also because these four numbers, together, unify the degree and decimal references when they are referenced around a unit circle as shown in figure (6) below.

Notice how number 199 is a continuation of number 19, and so is 341 a continuation of 161. In other words,  $199 - 19 = 180 = 341 - 161$ .

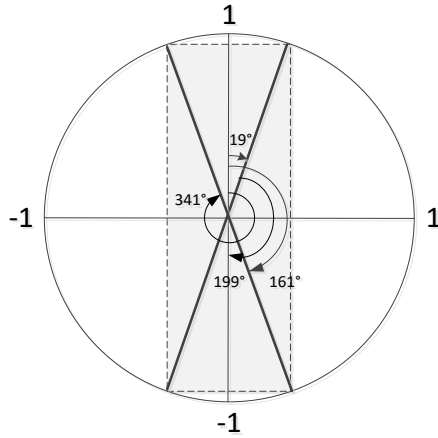


Fig.6: The angular relationships between the four numbers belonging to one complementary set. In this example,  $199^\circ$  is a continuation of  $19^\circ$  and so is  $341^\circ$  of  $161^\circ$ .

Dividing these numbers by 360 creates a normalized decimal reference for the set where the numbers now add up to 1 instead of 360. We can also give these decimal references signs that define their position on the unitary circle and maintain a balance of 0 around the whole circle. This is shown in figure (7) below for the set [(43, 317), (137, 223)].

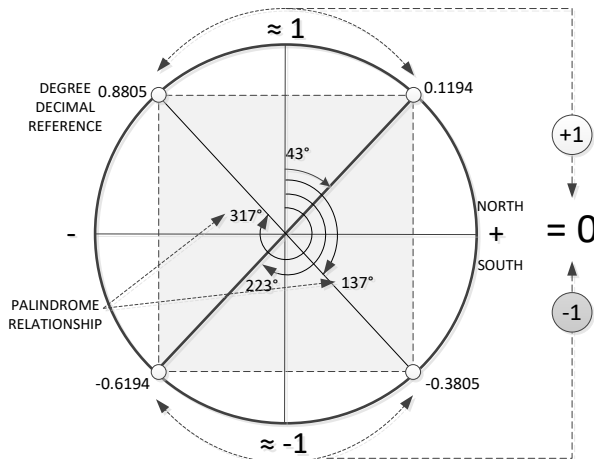


Fig.7: Normalized relationship for one complementarity set [(43, 317), (137, 223)] along with the decimal references and signs attributions.

For numbers beyond 360, the same complementary properties apply, however, the numbers will add up to  $n \times 360$  where  $n$  is an integer equal to 1 for the first 360 numbers, 2 for the next 360 numbers, and so on.

The quadrant relationships are better observed when we look at the trigonometric values of their reference angles as illustrated in the figure below.

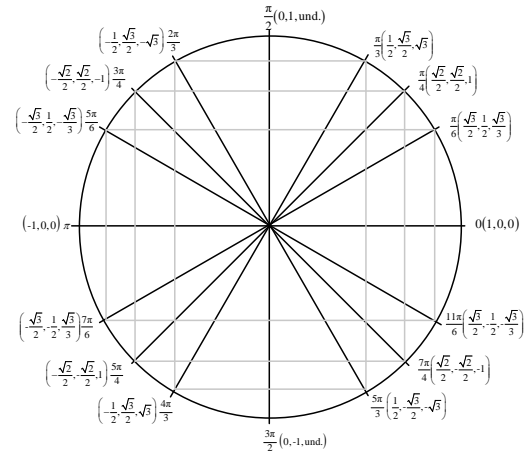


Fig.8: Trigonometric values for 3 different sets of quadrant-angles mirrored across the four quadrants, differing only with their  $\pm$  signs.

The values between the brackets are for the cosine, sine and tangent of the angles respectively (und. stands for undefined). Notice how they are mirrored across the quadrants, differing only by a  $\pm$  sign.

There are also complementary symmetries corresponding to central moduli numbers, continuing ad infinitum in multiples of 360 or 180 in a cyclic pattern, as shown below in figure (9).

All the above symmetric, as well as asymmetric, numeric relationships, are a consequence of the 2-dimensional geometric configuration of numbers and cannot easily be observed in a 1-dimensional linear form.

And while the symmetric relationships across the symmetry axis of the wheel may be explained via its geometry, the asymmetric ones cannot be easily explained, especially as they involve the

same axes of symmetry. More research is needed to find the roots of this symmetry-breaking behavior and how it is affected by the geometry.

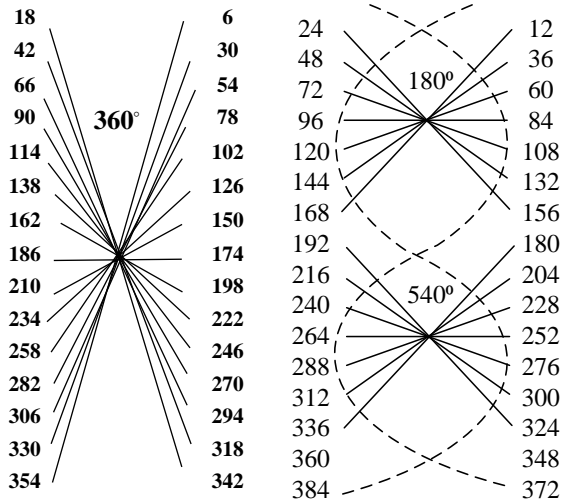


Fig.9: Numbers on the horizontal moduli complete to 360° (Left), and numbers on the vertical moduli complete to multiples of 180° (Right).

## V. CONCLUSION

The icositetragon-based configuration of numbers has been shown to have unique properties. The myriad of asymmetric and complementary relationships observed between the numbers in this configuration are clearly far from accidental nor can all be easily explained. The combined understanding of these relationships and their quadrant configuration provides a great insight into how numbers and geometry are interconnected and the rewards that may be gained from studying them as a whole. More in-depth research is needed to identify other hidden relationships as well as the origin of the asymmetric relationships between the various moduli.

## APPENDIX

A- Some of the unique properties of number 24 in relation to physics and higher dimensions:

Probably the most famous usage of number 24 is related to time as in the daily hours; with each day

divided into 24 hours of equal measure. (12 is also connected with time, being the number of months in one year.)

This 24-based division of the day has its origin in ancient Egypt<sup>5</sup> and has been implemented in timekeeping for centuries, from the western side of the world to the eastern one.

Also, of all numbers, 24 is the only nontrivial solution to the cannonball problem<sup>6</sup> that tries to find which squared pyramidal numbers add up to a perfect square number. The only solution is found to be:  $1^2 + 2^2 + 3^2 + \dots + 24^2 = 70^2$ , with 1, 2, 3... 24 being the sides of the squares.

Surprisingly, this unique property of number 24 is connected to Bosonic String theory and its perfect space of 26 (24+2) dimensions<sup>7</sup>, which is also related to the densest lattice packing of spheres in 24 dimensions, called the Leech Lattice<sup>8</sup>, where each sphere touches 196560 others.

Moreover, the lowest energy of a string is calculated to be:

$$\frac{1}{2}(1 + 2 + 3 + 4 + \dots \infty) = \frac{1}{2} \times -\frac{1}{12} = -\frac{1}{24}$$

(The infinite sum of positive integers is calculated, first by Euler<sup>9</sup>, to equal  $-\frac{1}{12}$ .)

Generally speaking, number 24 seems to thrive in higher dimensions, especially in the 4<sup>th</sup> dimension, where it appears much more frequently than in the 3<sup>rd</sup> dimension.

For example, among the six regular polytopes, the 4-dimensional counterparts of platonic solids, we find the 24-cell, made of 24 octahedral cells and vertices, and having a rotational symmetry of  $576 = 24^2$ . This polytope has no analogy in any other dimension, lower or higher.

Also, the tesseract, the 4-dimensional cube, has 24 2-dimensional square faces.

Moreover, in the 4<sup>th</sup> dimension, 24 is a *kissing number*<sup>10,11</sup>, being the maximum number of unit spheres all touching another unit sphere without overlapping, with the 24-cell residing at the center. (In the 3<sup>rd</sup> dimension, the kissing number is 12 while in the 8<sup>th</sup> dimension it is 240. So, there

seems to be some sort of connection between numbers 12 and 24, kissing spheres and dimensions.)

B- Proof that for any prime number  $p$ :

$$p^2 = k \times 24 + 1 \text{ (Always.)}$$

We know that every prime must come in the form of  $6k \pm 1$ . The factor  $k$  can be even or odd, so we write it as  $2m$  for even and  $2m+1$  for odd. Let us now substitute these in the main equation. We get four forms for  $p^2$  as follows:

$$(12m + 1)^2, (12m + 7)^2, (12m - 1)^2, \text{ and } (12m + 5)^2$$

When the squared terms are expanded, we get four terms that involve 24 times some factor of  $m$  plus 1, such as  $24(6m^2 + m) + 1$  and so on.

C- The Icositetragon wheel for numbers up to 1008.

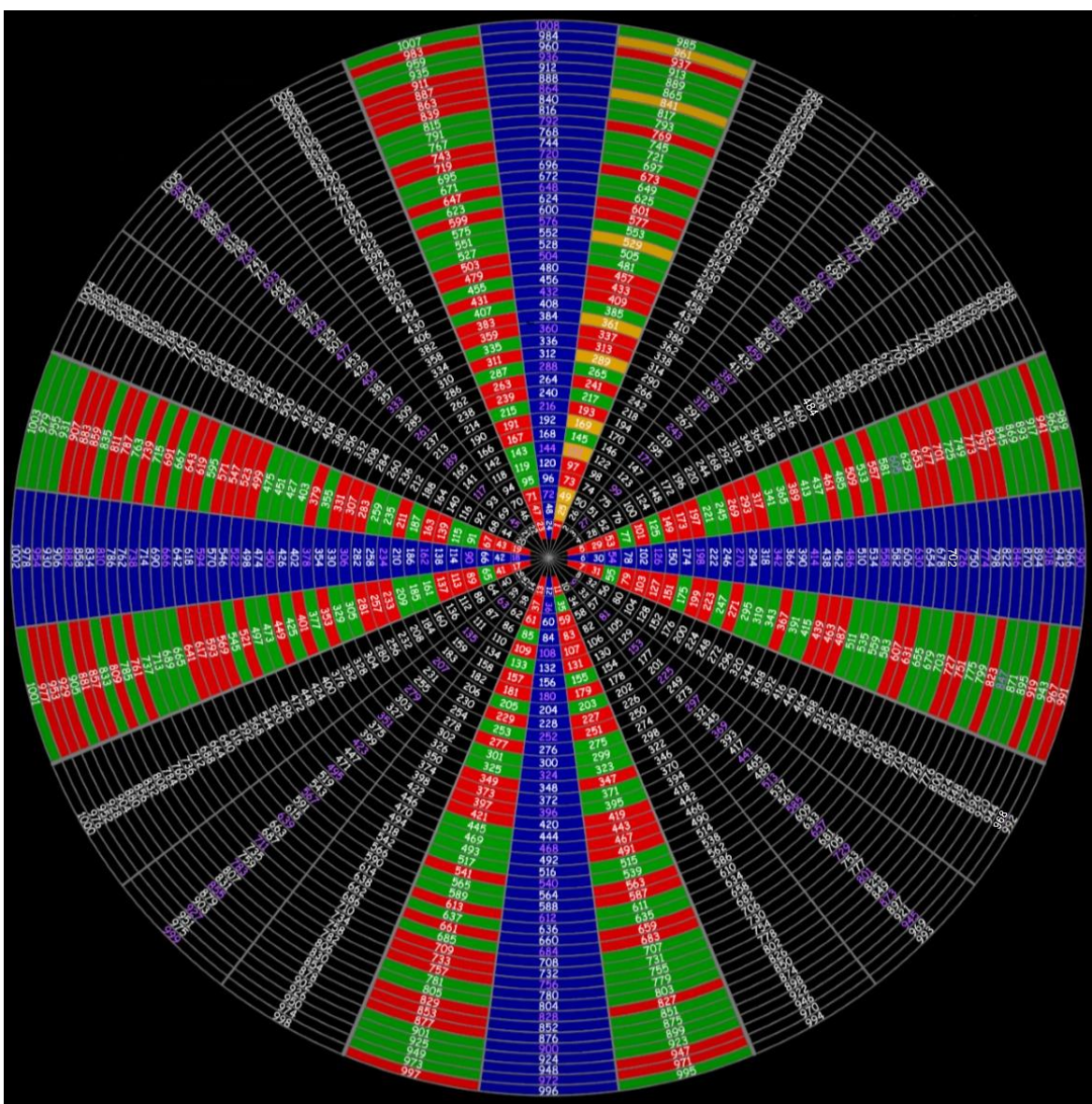
Legends:

Red: Prime Numbers.

Green: Quasi-Prime Numbers.

Orange: Prime Squared.

Black: All other numbers.



## REFERENCES

- [1] Robert E. Grant and Talal Ghannam. Accurate and Infinite Prime Prediction from Novel Quasi-Prime Analytical Methodology. arXiv:1903.08570 [math.NT] (2019).
- [2] Peter Plichta. God's Secret Formula: The Deciphering of the Riddle of the Universe and the Prime Number Code. Element Books Ltd (1998).
- [3] David Wells. Prime Numbers: The Most Mysterious Figures in Math. Wiley (2005).
- [4] John H. Mathews and Russell W. Howell. Complex Analysis: for Mathematics and Engineering, 6<sup>th</sup> edition. Jones and Bartlett, (2012).
- [5] Otto Neugebauer. The Egyptian Decans. Astronomy and History: Selected Essays. Springer, (1983).
- [6] Anglin, W. S. The Square Pyramid Puzzle. American Mathematical Monthly. Vol. 97, (1990).
- [7] D'Hoker, Eric & Phong, D. H. The geometry of string perturbation theory. Rev. Mod. Phys. American Physical Society. 60 (4), (1988).
- [8] Cohn, Henry and Kumar, Abhinav. Optimality and uniqueness of the Leech lattice among lattices. Annals of Mathematics. Vol. 170, (2009).
- [9] Lepowsky, J., Naihuan Jing, and Kailash C. Misra. Vertex operator algebras and the zeta function. Contemporary Mathematics. Vol. 248, (1999).
- [10] Odlyzko, A. M. and Sloane, N. J. A. New bounds on the number of unit spheres that can touch a unit sphere in  $n$  dimensions. J. Combin. Theory Ser. A 26, (1979).
- [11] Conway, John H. and Neil J.A. Sloane. Sphere Packings, Lattices and Groups (3rd ed.). Springer-Verlag, (1999).